

Complexes_Exemples_Cours_Partie1

December 10, 2016

```
In [1]: from sympy import*

In [2]: z,u, = symbols('z u')
        x = Symbol('x', real=True)
        y = Symbol('y', real=True)

In [3]: init_printing()

In [54]: def forme_algebrique(Z):
         return expand(Z, complex=True)

        def conjugue(Z):
            return Z.conjugate()

        def resoudre(E, inconnue):
            return solve(E, inconnue)

        def developper(exp):
            return collect(expand(exp), z)

        def factoriser(exp):
            return factor(exp)

        def simplifier(exp):
            return simplify(exp)
```

0.1 Exemple 2

```
In [5]: forme_algebrique((5 - 3 * I) * (2 + 3 * I))
```

Out[5]:

$$19 + 9i$$

```
In [6]: forme_algebrique((3 - 2 * I) ** 2)
```

Out[6]:

$$5 - 12i$$

```
In [7]: forme_algebrique((1 - I) ** 3)
```

```
Out[7]:
```

$$-2 - 2i$$

```
In [8]: forme_algebrique( 1 + 1 / I)
```

```
Out[8]:
```

$$1 - i$$

0.2 Exemple 3

```
In [12]: conjugue(2)
```

```
Out[12]:
```

$$2$$

```
In [13]: conjugue(-2 * I)
```

```
Out[13]:
```

$$2i$$

```
In [15]: conjugue( (-2 + 3 * I) / 6)
```

```
Out[15]:
```

$$-\frac{1}{3} - \frac{i}{2}$$

```
In [16]: conjugue( I ** 10)
```

```
Out[16]:
```

$$-1$$

```
In [17]: conjugue( ( - I ) ** 2013)
```

```
Out[17]:
```

$$i$$

0.3 Exemple 5

In [18]: `forme_algebrique(1 / (1 + I))`

Out [18]:

$$\frac{1}{2} - \frac{i}{2}$$

In [19]: `forme_algebrique((3 + 2 * I)/(2 - I))`

Out [19]:

$$\frac{4}{5} + \frac{7i}{5}$$

In [20]: `forme_algebrique(1/(3 - I) - 2 / (-1 + I))`

Out [20]:

$$\frac{13}{10} + \frac{11i}{10}$$

In [24]: `Equation_a = Eq(z - 3 * I, I * z + 1)`
`Equation_a`

Out [24]:

$$z - 3i = iz + 1$$

In [25]: `resoudre(Equation_a, z)`

Out [25]:

$$[-1 + 2i]$$

In [28]: `Equation_b = Eq(z * conjugue(z) + I * (z + 1) + conjugue(z))`
`Equation_b`

Out [28]:

$$z\bar{z} + i(z + 1) + \bar{z} = 0$$

In [30]: `resoudre(Equation_b, [z, conjugue(z)])`

Out [30]:

$$[{\{z: -1\}}, {\{\bar{z}: -i\}}]$$

In [31]: `Z = (x + I * y - I)/(x + I * y)`

In [32]: `re(Z)`

Out [32] :

$$\frac{x^2}{x^2 + y^2} + \frac{y(y - 1)}{x^2 + y^2}$$

In [33] : `im(Z)`

Out [33] :

$$-\frac{xy}{x^2 + y^2} + \frac{x(y - 1)}{x^2 + y^2}$$

0.4 Exemple 6

In [35] : `z + conjugue(z)`

Out [35] :

$$z + \bar{z}$$

In [36] : `re(z + conjugue(z))`

Out [36] :

$$2\Re z$$

In [37] : `im(z + conjugue(z))`

Out [37] :

$$0$$

$z + \bar{z}$ est un réel car il est égal à son conjugué.

In [38] : `conjugue(z + conjugue(z))`

Out [38] :

$$z + \bar{z}$$

$\frac{1}{z^3} - \frac{1}{\bar{z}^3}$ est un imaginaire pur car il non nul et égal à l'opposé de son conjugué

In [39] : `conjugue(1 / z**3 - 1 / conjugue(z) ** 3)`

Out [39] :

$$\frac{1}{\bar{z}^3} - \frac{1}{z^3}$$

$Z_3 = (2i + 3z)(3\bar{z} - 2i)$ est un réel car il est égal à son conjugué. On a d'ailleurs $Z_3 = (2i + 3z)\overline{2i + 3z} = |2i + 3z|^2$ et donc Z_3 est un réel positif.

In [41] : `conjugue((2 * I + 3 * z) * (3 * conjugue(z) - 2 * I))`

Out [41] :

$$(3z + 2i)(3\bar{z} - 2i)$$

0.5 Exemple 7

```
In [45]: Equation_1 = Eq(z ** 2, -8)
Equation_1
```

Out [45]:

$$z^2 = -8$$

```
In [43]: resoudre(Equation_1, z)
```

Out [43]:

$$\left[-2\sqrt{2}i, 2\sqrt{2}i\right]$$

```
In [46]: Equation_2 = Eq(9 * z ** 2 + 6 * z + 2)
Equation_2
```

Out [46]:

$$9z^2 + 6z + 2 = 0$$

```
In [47]: resoudre(Equation_2, z)
```

Out [47]:

$$\left[-\frac{1}{3} - \frac{i}{3}, -\frac{1}{3} + \frac{i}{3}\right]$$

```
In [49]: Equation_3 = Eq((5 * z + 5) / (1 - z), z)
Equation_3
```

Out [49]:

$$\frac{5z + 5}{-z + 1} = z$$

```
In [50]: resoudre(Equation_3, z)
```

Out [50]:

$$[-2 - i, -2 + i]$$

```
In [51]: a = Symbol('a', real=True)
b = Symbol('b', real=True)
```

```
In [55]: developper((z - 2) * (z ** 2 + a * z + b))
```

Out [55]:

$$-2b + z^3 + z^2(a - 2) + z(-2a + b)$$

In [57]: `resoudre([Eq(a - 2, -1), Eq(-2*a + b, -1), Eq(-2*b, -2)], [a, b])`

Out [57]:

$$\{a : 1, b : 1\}$$

In [59]: `Equation_4 = Eq(z ** 3 - z ** 2 - z - 2)`
`Equation_4`

Out [59]:

$$z^3 - z^2 - z - 2 = 0$$

In [60]: `resoudre(Equation_4, z)`

Out [60]:

$$\left[2, -\frac{1}{2} - \frac{\sqrt{3}i}{2}, -\frac{1}{2} + \frac{\sqrt{3}i}{2} \right]$$

0.6 Exemple 8 Métropole juin 2014

In [62]: `Equation_A = Eq(z ** 2 + 4 * z + 16)`

In [64]: `resoudre(Equation_A, z)`

Out [64]:

$$\left[-2 - 2\sqrt{3}i, -2 + 2\sqrt{3}i \right]$$

In [68]: `a = 2 * (cos(pi/3) + I * sin(pi/3))`
`a`

Out [68]:

$$1 + \sqrt{3}i$$

In [71]: `forme_algebrique(a ** 2)`

Out [71]:

$$-2 + 2\sqrt{3}i$$

In [73]: `Equation_B = Eq(z ** 2, -2 + 2 * I * sqrt(3))`
`Equation_B`

Out [73]:

$$z^2 = -2 + 2\sqrt{3}i$$

In [76]: Solution_B = resoudre(Equation_B, z)
Solution_B

Out [76]:

$$\left[-\sqrt{-2+2\sqrt{3}i}, \sqrt{-2+2\sqrt{3}i}\right]$$

In [80]: Solution_B[0]

Out [80]:

$$-\sqrt{-2+2\sqrt{3}i}$$

In [81]: forme_algebrique(Solution_B[0])

Out [81]:

$$-1 - \sqrt{3}i$$

In [82]: Solution_B[1]

Out [82]:

$$\sqrt{-2+2\sqrt{3}i}$$

In [83]: forme_algebrique(Solution_B[1])

Out [83]:

$$1 + \sqrt{3}i$$

In [85]: Equation_C = Eq(z ** 4 + 4 * z ** 2 + 16)
Equation_C

Out [85]:

$$z^4 + 4z^2 + 16 = 0$$

In [86]: resoudre(Equation_C, z)

Out [86]:

$$\left[-1 - \sqrt{3}i, -1 + \sqrt{3}i, 1 - \sqrt{3}i, 1 + \sqrt{3}i\right]$$